New Specification

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Further Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

[SFM11]

Assessment

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1. (i) M1 Trying to multiply to obtain $\mathbf{A}^2$

W1 Correct value of $\mathbf{A}^2 \begin{pmatrix} 4 & 7 \\ 0 & 25 \end{pmatrix}$

M1 Trying to calculate $7\mathbf{A} - 10\mathbf{I}$

W1 Correct value found as $\begin{pmatrix} 4 & 7 \\ 0 & 25 \end{pmatrix}$

(ii) M1 Recognising that $\mathbf{A}^3 = \mathbf{A}(7\,\mathbf{A} - 10\mathbf{I})$

W1 Expanding to give $7\mathbf{A}^2 - 10\mathbf{A}$

M1 Substituting $\mathbf{A}^2 = 7\mathbf{A} - 10\mathbf{I}$

W1 Correct answer found as $39\mathbf{A} - 70\mathbf{I}$

<u>Alternative Solution</u>

MW1 Calculates $\mathbf{A}^3 = \begin{pmatrix} 8 & 39 \\ 0 & 125 \end{pmatrix}$

M1 Tries to equate with $m\mathbf{A} + n\mathbf{I}$

W1 Obtains $m = 39$

W1 Obtains $n = -70$

2. MW1 Stating sum of roots correctly as – p

MW1 Stating product of roots correctly as q

M1 Trying to find $\frac{1}{\alpha} + \frac{1}{\beta}$

W1 Correct value found as $-\frac{p}{q}$

MW1 Correct product of $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ found as $\frac{1}{q}$

MW1 Correct equation set up (may be left with fractional coefficients)

3. (i) M1 Trying to multiply v and w

W1 Correct answer found as $11 - 2i$

(ii) M1 Knowing to multiply by $\frac{1 + 2i}{1 + 2i}$

MW1 Correct numerator

MW1 Correct denominator

(No penalty if not fully simplified to $-1 + 2i$)

(iii) MW1 Stating that locus is a perpendicular bisector

MW1 Line clearly marked as perp. to line joining points (1, –2) and (3, 4)

MW1 Line clearly marked as passing through mid-point i.e. (2, 1)

4. (a) MW1 States “reflection”

MW1 States “in y-axis” or equivalent

(b) (i) M1 Tries to set up matrix equation

MW1 Expands to get 1 pair of equations

MW1 Expands to get 2nd pair of equations

MW1 Obtains 2 correct pairs of values

W1 States correct matrix

Notes

[A] Multiplies matrices in wrong order – can award

M1 – tries to set up matrix equation

MW1(ft) – expands to get 1 pair of equations

i.e. max of 2/5

Alternative Solution

M1 Tries to set up matrix equation

MW1 Finds correct inverse

M1 Tries to multiply by inverse

W1 Multiplies in correct order

W1 States correct matrix

Notes

[A] Multiplies matrices in wrong order – can award

M1 – tries to set up matrix equation

MW1 – finds correct inverse

i.e. max of 2/5

(ii) M1 Trying to find area of triangle OAB

W1 Obtains correct answer of 2.5

M1 Trying to use Area Δ OA′B′ = det $\mathbf{M} \times 2.5$

MW1 Obtains correct area

Notes

[A] Carries through incorrect $\mathbf{M}$ from (i) – can award

M1 W1 M1(ft) MW0

i.e. max of 3/4

5. (i) MW1 Correct answer only

MW1 Correct reason

(ii) M1 Knows format of factorised equation is product of 2 linear terms and a quadratic

W1 Expands 2 known factors to obtain $z^2 - 2z + 5$

M1 Tries to find quadratic factor (long division or recognition)

W2 Correct factor found as $z^2 + 9$ (W1 for each of p, q)

MW2 Obtains 2 correct solutions as $\pm 3i$

Notes

[A] Carries through incorrect root from (i) – can award

M1 W0 M1 W0 MW0

i.e. max of 2/7

6. (i) M1 Tries to expand determinant

W1 Correctly expands their answer (allow ft for minor calculation error)

W1 Simplified to correct answer

(ii) (a) MW1 Correctly substitutes $a = 1 \Rightarrow$ determinant = 38

W1 Correct statement "one unique solution" oe

(b) MW1 Substitutes $a = 2$ to set up 3 equations

M1 Tries to eliminate to obtain one equation in terms of 2 variables

W1 Eliminates correctly

MW1 Eliminates to obtain another equation in terms of the same 2 variables

MW1 Correct statement "no solution"

7. (i) M1 Tries to find 2 suitable vectors

W1 Correct value for $\overrightarrow{\mathbf{QR}}$

MW1 Correct value for $\overrightarrow{\mathbf{QS}}$ (or $\overrightarrow{\mathbf{RS}}$)

M1 Recognises that one is a multiple of other

W1 Correct statement

(ii) M1 Tries to set up an equation of the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$

W1 Correct equation

(iii)	M1	Tries to use scalar product = 0 (at least one vector must be correct)
	W1	Correct expansion of their product (allow ft)
	MW2	2 correct answers found
(iv)	M1	Tries to use vector product to find perpendicular vector
	MW1	Finds vector $\overrightarrow{\mathbf{QT}} = 2\mathbf{i} - \mathbf{k}$
	W1	Obtains correct answer
	M1	Tries to set up $\mathbf{r} \cdot \mathbf{n} = d$
	M1	Substitutes position vector of Q, R or T into $\mathbf{r} \cdot \mathbf{n} = d$ (allow ft for their $\mathbf{n}$)
	W1	Finds correct value of d
	W1	States correct equation of plane
8. (i)	M1	Correct method for finding modulus
	W1	Correct value of modulus
	M1	Correct method for finding argument
	W1	Correct value of argument (do not penalise if in degrees)
(ii)	M1	Correct method for finding complex number
	W1	Correct answer
(iii)	MW1	Correct position shown for z_1
	MW1	Correct position marked for z_2
	MW1	Correct position drawn for $z_1 + z_2$
(iv)	MW1	Finds correct half-angle in parallelogram (θ)
	M1	Tries to find correct angle for arg $(z_1 + z_2)$ (α)
	W1	Obtains correct value of α
	MW1	Finds correct answer (must have evidence of these numbers on diagram or elsewhere in question)

9.	(i)	M1	Tries to set up 3 equations in λ, μ
		W2	Finds 3 correct equations (P1 each incorrect or missing equation)
		M1	Tries to eliminate one variable
		W1	Finds correct value of either λ or μ
		W1	States correct coordinates for C – do not accept $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ or $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$
	(ii)	M1	Tries to substitute components of BD into equation of plane
		W1	Substitutes correctly
		MW1	Finds correct value of λ
		W1	States correct coordinates for D – do not accept $\begin{pmatrix} 2 \\ -4 \\ 7 \end{pmatrix}$ or $2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$
	(iii)	M1	Tries to use correct formula for volume of tetrahedron
		MW3	Substitutes each of 3 correct vectors into formula (allow ft of their C, D)
		MW1	Correct value for vector product
		MW1	Correct value for scalar product
		MW1	Correct volume

			AVAILABLE MARKS
1	**(i)** $\mathbf{A}^2 = \begin{pmatrix} 2 & 1 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 5 \end{pmatrix}$	M1	
	$= \begin{pmatrix} 4 & 7 \\ 0 & 25 \end{pmatrix}$	W1	
	$7\mathbf{A} - 10\mathbf{I} = \begin{pmatrix} 14 & 7 \\ 0 & 35 \end{pmatrix} - \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$	M1	
	$= \begin{pmatrix} 4 & 7 \\ 0 & 25 \end{pmatrix}$	W1	
	(ii) $\mathbf{A}^3 = \mathbf{A} \times \mathbf{A}^2$		
	$= \mathbf{A}(7\mathbf{A} - 10\mathbf{I})$	M1	
	$= 7\mathbf{A}^2 - 10\mathbf{A}$	W1	
	$= 7(7\mathbf{A} - 10\mathbf{I}) - 10\mathbf{A}$	M1	
	$= 39\mathbf{A} - 70\mathbf{I}$	W1	8
2	$\alpha + \beta = -\text{p}$	MW1	
	$\alpha\beta = \text{q}$	MW1	
	$\Rightarrow \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha + \beta}{\alpha\beta}$	M1	
	$= -\dfrac{\text{p}}{\text{q}}$	W1	
	and $\dfrac{1}{\alpha\beta} = \dfrac{1}{\text{q}}$	MW1	
	Hence the equation is $x^2 + \dfrac{\text{p}}{\text{q}}x + \dfrac{1}{\text{q}} = 0$		
	$\Rightarrow \text{q}x^2 + \text{p}x + 1 = 0$	MW1	6

AVAILABLE MARKS

3 **(i)** $(3 + 4i)(1 - 2i)$

$= 3 + 4i - 6i + 8$ M1

$= 11 - 2i$ W1

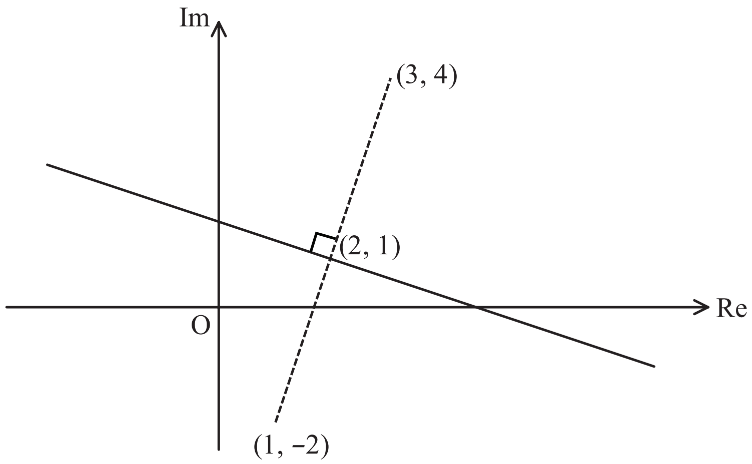
(ii) $\dfrac{3 + 4i}{1 - 2i} \times \dfrac{1 + 2i}{1 + 2i}$ M1

$= \dfrac{3 + 4i + 6i - 8}{1 - 2i + 2i + 4}$

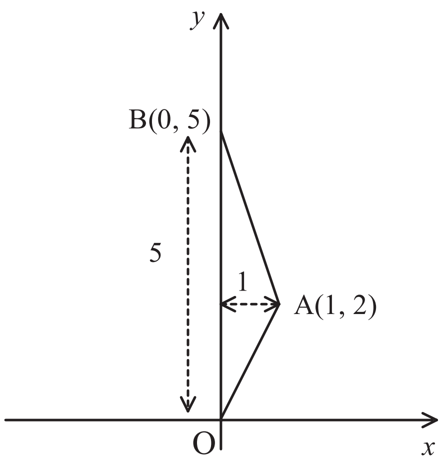
$= \dfrac{-5 + 10i}{5}$

$= -1 + 2i$ MW2

(iii) Perpendicular bisector of the line joining the points (1, –2) and (3, 4) MW1



MW2 8

			AVAILABLE MARKS
4	**(a)** Reflection in the y-axis	MW2	
	(b) (i) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ 4 & 10 \end{pmatrix}$	M1	
	$\Rightarrow a + 2b = 9 \qquad c + 2d = 4$	MW1	
	$5b = 20 \qquad 5d = 10$	MW1	
	$\Rightarrow \qquad b = 4 \qquad d = 2$		
	and $\qquad a = 1 \qquad c = 0$	MW1	
	$\Rightarrow \mathbf{M} = \begin{pmatrix} 1 & 4 \\ 0 & 2 \end{pmatrix}$	W1	
	Alternative Solution		
	$\mathbf{M} \begin{pmatrix} 1 & 0 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ 4 & 10 \end{pmatrix}$	M1	
	Inverse of $\begin{pmatrix} 1 & 0 \\ 2 & 5 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 5 & 0 \\ -2 & 1 \end{pmatrix}$	MW1	
	$\Rightarrow \mathbf{M} \begin{pmatrix} 1 & 0 \\ 2 & 5 \end{pmatrix} \times \frac{1}{5} \begin{pmatrix} 5 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ 4 & 10 \end{pmatrix} \times \frac{1}{5} \begin{pmatrix} 5 & 0 \\ -2 & 1 \end{pmatrix}$	M1 W1	
	$\Rightarrow \mathbf{M} = \frac{1}{5} \begin{pmatrix} 5 & 20 \\ 0 & 10 \end{pmatrix}$		
	$\Rightarrow \mathbf{M} = \begin{pmatrix} 1 & 4 \\ 0 & 2 \end{pmatrix}$	W1	
	(ii) 		
	Area ΔOAB $= \frac{1}{2} \times 5 \times 1$	M1	
	$= 2.5$	W1	
	Area ΔOA'B' = det $\mathbf{M} \times 2.5$	M1	
	$= 2 \times 2.5$		
	$= 5$ square units	MW1	11

				AVAILABLE MARKS

5 **(i)** Another root is $1 - 2\text{i}$ MW1

This can be stated since non-real roots of polynomial equations with real coefficients occur in conjugate pairs. MW1

(ii) $[z - (1 + 2\text{i})]$ and $[z - (1 - 2\text{i})]$ are 2 of the factors M1

$$[z - (1 + 2\text{i})][z - (1 - 2\text{i})]$$
$$= z^2 - (1 + 2\text{i})\,z - (1 - 2\text{i})\,z + (1 + 2\text{i})(1 - 2\text{i})$$
$$= z^2 - 2z + 5 \quad \text{W1}$$

$$z^4 - 2z^3 + 14z^2 - 18z + 45 = (z^2 - 2z + 5)(z^2 + pz + q)$$
$$= (z^2 - 2z + 5)(z^2 + 9) \quad \text{M1 W2}$$

$$z^2 + 9 = 0$$
$$\Rightarrow z = \pm 3\text{i}$$
and $z = 1 \pm 2\text{i}$ MW2 — 9

6 **(i)** $\begin{vmatrix} 1 & 1 & c \\ c+1 & 6 & 9 \\ 5 & -1 & 4 \end{vmatrix} = 1(24 + 9) - 1(4(c+1) - 45) + c(-(c + 1) - 30)$ M1

$= 33 - 4c - 4 + 45 - c^2 - c - 30c$ W1

$= -c^2 - 35c + 74$ W1

(ii) **(a)** $c = 1 \Rightarrow$ determinant $= 38$ MW1

Since this is non-zero, the equations have one unique solution. W1

(b) $c = 2 \Rightarrow$ determinant $= 0$

Since this is zero, the equations do not have one unique solution.

$x + y + 2z = 0$ ① MW1
$3x + 6y + 9z = -7$ ②
$5x - y + 4z = 1$ ③

① + ③ $\Rightarrow 6x + 6z = 1$ M1 W1

② + 6③ $\Rightarrow 33x + 33z = -1$ MW1

i.e. $x + z = \dfrac{1}{6}$ and $x + z = -\dfrac{1}{33}$

Since these are contradictory, the equations have no solution. MW1 — 10

				AVAILABLE MARKS
7	(i)	$\overrightarrow{\mathbf{QR}} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$	M1 W1	
		$\overrightarrow{\mathbf{QS}} = 3\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$	MW1	
		$\Rightarrow \overrightarrow{\mathbf{QS}} = 3\overrightarrow{\mathbf{QR}}$	M1	
		Hence QS and QR are parallel and since Q is also common, the points are collinear.	W1	
	(ii)	$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ (or equivalent)	M1 W1	
	(iii)	$(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \cdot (-\mathbf{i} + p\mathbf{j} + p^2\,\mathbf{k}) = 0$	M1	
		$\Rightarrow -1 + 3p - 2p^2 = 0$	W1	
		$\Rightarrow 2p^2 - 3p + 1 = 0$		
		$\Rightarrow (2p - 1)(p - 1) = 0$		
		$\Rightarrow p = \frac{1}{2}, 1$	MW2	
	(iv)	Perpendicular vector to plane is $\overrightarrow{\mathbf{QR}} \times \overrightarrow{\mathbf{QT}}$	M1	
		$\overrightarrow{\mathbf{QT}} = 2\mathbf{i} - \mathbf{k}$	MW1	
		$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -2 \\ 2 & 0 & -1 \end{vmatrix} = -3\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}$	W1	
		Taking perpendicular vector as $\mathbf{n} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$		
		$\Rightarrow \mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = d$	M1	
		$\Rightarrow (2\mathbf{i} - \mathbf{j} + 7\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = d$	M1	
		$\Rightarrow d = 2 - 1 + 14 = 15$	W1	
		$\Rightarrow \mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = 15$	W1	18

8 **(i)** $|z_1| = \sqrt{2+2}$ M1

$= 2$ W1

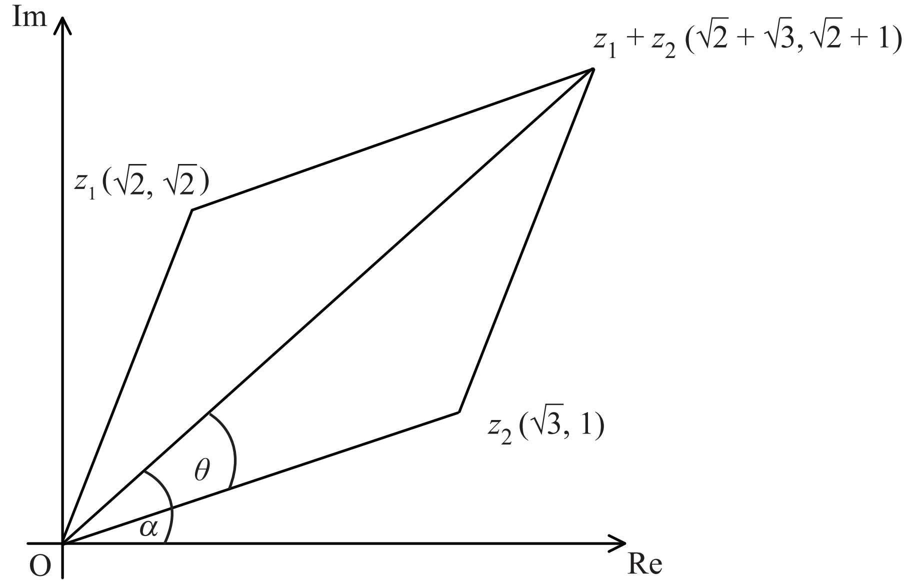
$\arg z_1 = \tan^{-1}\frac{\sqrt{2}}{\sqrt{2}}$ M1

$= \frac{\pi}{4}$ W1

(ii) $z_2 = 2\cos\frac{\pi}{6} + 2\sin\frac{\pi}{6}\,\mathrm{i}$ M1

$= \sqrt{3} + \mathrm{i}$ W1

(iii)



MW2

MW1

(iv) $|z_1| = |z_2| \Rightarrow$ the shape formed by O, z_1, z_2 and $z_1 + z_2$ is a rhombus

Hence $\theta = \frac{1}{2}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\pi}{24}$ MW1

$\Rightarrow \alpha = \frac{\pi}{6} + \frac{\pi}{24}$ M1

$= \frac{5\pi}{24}$ W1

$\Rightarrow \tan\left(\frac{5\pi}{24}\right) = \frac{\sqrt{2}+1}{\sqrt{2}+\sqrt{3}}$ MW1 13

AVAILABLE MARKS

				AVAILABLE MARKS
9	(i)	$1 + \lambda = 2\mu$ ①	M1	
		$2 - 6\lambda = -7 + 3\mu$ ②		
		$3 + 2\lambda = 4 + \mu$ ③	W2	
		③ $-2 \times$ ① $\Rightarrow 1 = 4 - 3\mu$	M1	
		$\mu = 1$	W1	
		$\lambda = 1$		
		Check with ② $\Rightarrow 2 - 6 = -7 + 3$ (correct)		
		C $(2, -4, 5)$	W1	
	(ii)	$6x + y = 8$		
		$\Rightarrow 6(4 + 2\gamma) + (-1 + 3\gamma) = 8$	M1 W1	
		$24 + 12\gamma - 1 + 3\gamma = 8$		
		$\gamma = -1$	MW1	
		D $(2, -4, 7)$	W1	
	(iii)	Volume $= \frac{1}{6}\left\|\left(\overrightarrow{\mathbf{CA}} \times \overrightarrow{\mathbf{CB}}\right) \cdot \overrightarrow{\mathbf{CD}}\right\|$	M1	
		$= \frac{1}{6}\left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 6 & -2 \\ 2 & 3 & 1 \end{vmatrix} \cdot 2\mathbf{k} \right\|$	MW3	
		$= \frac{1}{6}\left\|(12\mathbf{i} - 3\mathbf{j} - 15\mathbf{k}) \cdot 2\mathbf{k}\right\|$	MW1	
		$= \frac{1}{6} \times 30$	MW1	
		$= 5$ cubic units	MW1	17
		Alternative solution		
		Volume $= \frac{1}{6}\left\|\left(\overrightarrow{\mathbf{CA}} \times \overrightarrow{\mathbf{CB}}\right) \cdot \overrightarrow{\mathbf{CD}}\right\|$	M1	
		$= \frac{1}{6}\left\| \begin{vmatrix} -1 & 6 & -2 \\ 2 & 3 & 1 \\ 0 & 0 & 2 \end{vmatrix} \right\|$	MW3	
		$= \frac{1}{6}\left\|-30\right\|$	MW1	
		$= \frac{1}{6}(30)$	MW1	
		$= 5$ cubic units	MW1	
			Total	**100**